Fluid Mechanics - Course 123

COMPRESSIBLE FLOW

Flow of compressible fluids in a pipe involves not only change of pressure in the downstream direction but also a change of both density of the fluid and the velocity of flow. The situation is further complicated by the fact that heat may be transferred through the walls of the pipe.

If the pipe is well insulated the heat transfer may be negligible and the changes therefore adiabatic (but not, of course, isentropic). In short pipes where no specific provision is made for heat transfer the conditions may approximate to adiabatic. On the other hand, for flows at low velocities in long, uninsulated pipes, an appreciable amount of heat may be transferred through the pipe walls and if the temperatures inside and outside the pipe are similar the flow may be approximately isothermal. This is so, for example in long compressed-air pipelines and in low-velocity flows generally.

ADIABATIC FLOW

Gas flow through a pipe or constant area duct is considered subject to the following assumptions:

1) Perfect gas (constant specific heats)
2) Steady, one-dimensional flow
3) Adiabatic flow (no heat transfer through the walls)
4) Friction factor is constant over the length of the conduit
5) Changes in elevation are insignificant in relation to frictional effects.
6) No work is added to, or extracted from the flow

The controlling equations are those of continuity, energy, momentum and state.

From 1st Law of thermodynamics, in a system where the mass is constant, the amount of heat supplied to the system is equal to the increase in energy of the system plus all the energy which leaves the system as work is done.

Thus $\Delta Q = \Delta E + \Delta W$

ΔQ = heat added
ΔW = work done
ΔE = change in Σ, kinetic, potential and internal energies

If we consider work done by a fluid ΔW is not the only work done. Work is done in overcoming the pressure forces, this called 'flow-work'.

$$\text{Flow Work} = P_2A_2ds_2 - P_1A_1ds_1$$

ds is incremental distance

If we consider the flow work in specific energy terms i.e. energy/unit mass we get flow work

$$= \frac{P_2A_2ds_2}{dm} - \frac{P_1A_1ds_1}{dm}$$

$$= \frac{P_2}{\varrho} - \frac{P_1}{\varrho}$$

Thus the whole equation now becomes:

$$Q = \left(\frac{P_2}{\varrho_2} + \frac{V_2{}^2}{2} + gZ_2\right) - \left(\frac{P_1}{\varrho_1} + \frac{V_1{}^2}{2} + gZ_1\right) + U_2 - U_1 + W^1$$

Q is the heat added to the fluid per unit mass
W^1 represents the work done by the fluid per unit mass

$U + \frac{P}{\varrho}$ may be written as specific enthalpy 'h'

Thus $h + \frac{V^2}{2} + gZ$ = constant along a streamline if no heat is added or subtracted from the fluid and no mechanical work is done.

If the fluid is a perfect gas h = CpT.

If there is no change in elevation then constant $ho = h + \frac{V^2}{2}$ where ho is the total head or stagnation enthalpy

$$Qm = \varrho AV$$

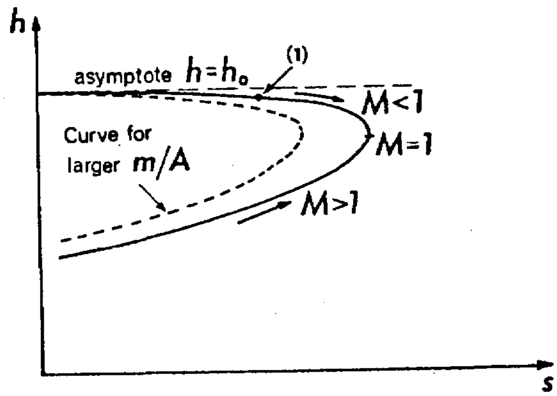
$$\text{Thus } ho = h + 0.5 \quad \frac{Qm}{\varrho A}^{2}$$

For given values of Qm, A and the stagnation enthalpy ho, curves of h against ϱ could be plotted using the previous equation.

A more significant relationship exists between h and specific entropy S. Entropy, like h & ϱ is a function of state and so may be determined from values of h & ϱ.

For a perfect gas $S - S_1 = C_V \log \left[\left(\frac{h}{h_1}\right)\left(\frac{P_1}{\varrho}\right)^{\gamma - 1}\right]$

Starting from a specified state, point 1 in the diagram, the curve of h against S traces the states through which the substance must pass in an adiabatic process.



This curve is called a Fanno curve (Gino Fanno Italian Eng.) All Fanno curves show a maximum value of S. It may be shown that the specific entropy is a maximum when the Mach number is unity.

Mach No. M = ratio - velocity of the fluid to the velocity of sound in the fluid

$$= \frac{V}{a} \quad \text{(dimensionless)}$$

The upper part of the curve which approaches the stagnation enthalpy ho corresponds to subsonic flow and the lower branch to supersonic flow.

Since, for adiabatic conditions the entropy cannot decrease, friction acts to increase the Mach no. in subsonic flow and reduce the Mach no. in supersonic flow. As friction involves a continual increase in entropy, sonic velocity can only be reached at the exit of the pipe, if at all.

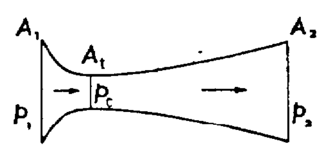
If sonic velocity is to be reached in a particular pipe then, for given inlet conditions and outlet pressure, a certain length is necessary. If the actual length is less than this "limiting" value, sonic conditions are not reached.

If the length of the pipe is increased beyond the limiting value, an initially subsonic flow will be choked, that is, the rate of flow will be reduced so as again to give sonic conditions of the outlet. An initially supersonic flow will also be adjusted to give sonic conditions at the exit; a normal shock will form near the end of the pipe and the resulting subsonic flow will accelerate to sonic conditions at the exit. Further increase in length would cause the shock to move towards the inlet of the pipe and then into the nozzle producing the supersonic flow so that the flow would become entirely subsonic in the pipe. Thus the maximum flowrate occurs when M = 1.

The following table shows how properties vary for adiabatic flow with friction.

Property		Subsonic Flow	Supersonic Flow
Mach No.	M	Increases	Decreases
Specific enthalpy	h	Decreases	Increases
Velocity	V	Increases	Decreases
Density	ϱ	Decreases	Increases
Temperature	T	Decreases	Increases
Pressure	P	Decreases	Increases
Re = $\frac{VD\varrho}{\mu}$	T	Increases	Decreases
Stagnation temp	To	Constant	Constant

At high velocities, the rate at which friction dissipates mechanical energy is large and supersonic flow in a pipe is generally better avoided. If supersonic flow is subsequently required the gas may be expanded in a convergent - divergent nozzle, where isentropic flow occurs.



In the convergent section of a nozzle, the initial flow is subsonic and the velocity is accelerated as the gas approaches the throat which is the area of minimum cross section. The maximum velocity at the throat occurs when there is sonic velocity and this is known as the critical condition. The pressure ratio to achieve this is called the critical pressure ratio and is the ratio of the pressure at the throat in relation to the stagnation Pressure. i.e. the pressure with no flow.

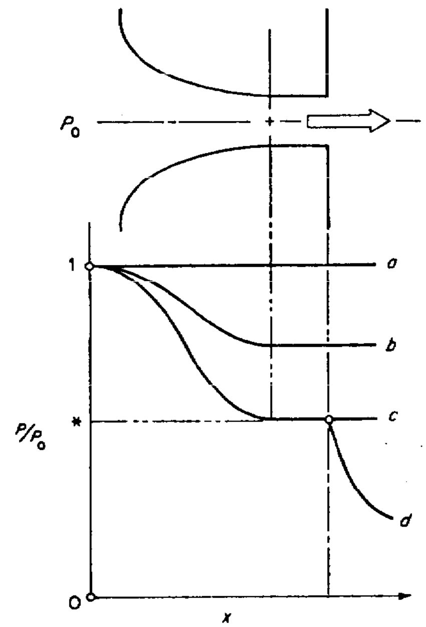
$$\frac{P_c}{P_o} = \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma - 1}} \qquad (\gamma = \text{ratio of specific heats } \frac{C_p}{C_v})$$

γ for air = 1.4 and critical pressure ratio is 0.528, whereas for superheated steam $\gamma \simeq 1.3$ and critical press ratio 0.546.

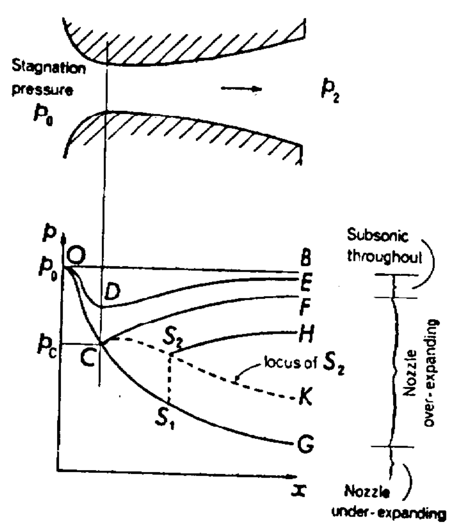
If the pressure of a vapour falls below saturation pressure the vapour no longer performs at a perfect gas. γ changes and the equation for $\frac{P_e}{P_o}$ may no longer be applied.

Thus the maximum flow rate is a function of the critical pressure ratio and the throat area. No matter how the downstream pressure is reduced this flow rate may not be exceeded. The nozzle is now in the "choked" condition. For the flow to alter in response to a change in downstream pressure a rarefaction wave has to travel upstream to the initial condition. However the velocity of the rarefaction, is at maximum, only that of sonic velocity and obviously if the flow at the throat is already sonic then there is no way that the upstream pressure may be sensitive to the downstream pressure.

The pressure distribution in a convergent nozzle may be seen in the diagram. At external pressure P_a there is no ΔP and no flow. At pressure P_b the flow is subsonic at the throat. At pressure P_c, when the critical pressure ratio is reached, the flow is sonic. As the outside pressure is further decreased the flow through the nozzle remains constant and has an exit pressure equal to P_c. Further expansion takes place outside the nozzle to pressure P_d. In this situation, when there is no further response by flow rate, to the lowering of actual pressure, the nozzle is said to be choked.



Pressure distributions in a converging nozzle.



In the divergent section the pressure downstream from the throat varies with outlet pressure. Consider P_o to be fixed but the outlet pressure P_2 is variable. Consider $P_2 = P_o$ then there would be no flow and pressure through the nozzle would be P_o represented by line OB. Reduction in P_2 causes a reduction in the pressure at the end of the nozzle and as the velocity at the throat is nowhere near sonic this reduction is seen upstream from the throat and a pressure line ODE is obtained.

Further decrease of external pressure increases the velocity at the throat of the nozzle and reduces the throat pressure until P_2 corresponds to point F. At this point the throat velocity is then sonic and the pressure distribution is represented OCF.

P_2 is further reduced the conditions in the convergent part of the nozzle remain unchanged and the upstream pressure distribution follows the single curve OC. If the external pressure P_2 exactly correspond to point F then there is an adiabatic compression following curve CF and the velocity downstream of the throat is entirely subsonic. Only this value of the external pressure, however, allows a compression from the point C. It should be noted that continuous isentropic expansion at supersonic velocity is only possible according to the curve CG.

In a nozzle designed to produce supersonic flow the smooth expansion OCG is the ideal and the ratio $\frac{P_o}{P_G}$ is the "design pressure ratio" of the nozzle.

It is apparent that there exists a range of exit pressures for which isentropic flow through a convergent-divergent is not possible. Experiment shows that under these conditions the flow in the nozzle undergoes, at some point in the diverging section, an abrupt change from supersonic to subsonic velocity. This change is accompanied by large and abrupt rises in pressure, density and temperature. The zone in which these changes occur is so thin that for calculations outside the zone it may be considered as a flow discontinuity. This discontinuity is called a "normal shock wave" (i.e. a shock wave perpendicular to the flow direction). The thickness of the wave is around 10^{-6} m. In this wave the gradients of velocity and density are so steep that viscous action, heat conduction and mass diffusion are all appreciable with the result that the flow undergoes a large entropy increase as it passes through the wave.

The normal shock wave is only a special case of the broader dass of flow discontinuities called "oblique shock waves" that are found in most supersonic flows, both internal (in ducts, pipes, jet engine intakes and compressors) and external (over the surfaces of wings, etc.)

For external pressures, at the nozzle, between F & G flow cannot take place without the formation of a shock wave and consequent dissipation of energy. In such circumstances the nozzle is said to be "over expanding".

P_2 is reduced slightly below F a normal shock wave is formed downstream of the throat. The pressure follows curve CG only as far as S, then there is an abrupt rise of pressure through the shock (S_1S_2) and then subsonic deceleration of the flow with rise of pressure to P_H.

As the external pressure is lowered the shock moves further from the throat until when $P_2 = P_k$, the point S_1 has moved to G and S_2 has moved to k. For exit press less than P_k, the flow within the entire divergent section of the nozzle is supersonic and follows line CG.

If P_2 lies between k and G a compression must occur outside the nozzle to raise the pressure from P_G to the external pressure. This compression involves oblique shock waves which cannot be dealt with in 'one dimensional' flow terms.

When the external pressure is below P_G, the nozzle is said to be under expanding and the expansion to pressure P_2 takes place outside the nozzle again by oblique expansion.

ASSIGNMENT

1) Discuss how flow is affected by pressure in a convergent nozzle.

2) Discuss how supersonic flow may be obtained from a convergent divergent nozzle.

3) Why is it impossible to obtain supersonic flow in a convergent nozzle.

4) Why do we need to use nozzles in engineering.

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